Real and Financial Options

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Introduction

- This paper builds a bridge between the real decision of firms and prices of an extremely important financial product: equity options
- Option pricing received enormous attention
    - Many influential models developed by both academics and practitioners
    - ... reduced form stock, volatility, jumps dynamics
    - ... aimed to fit the observed prices
- **Silent on the effect of firm characteristics and fundamental determinants of the cross section of option prices**
- I fill this gap by developing a production based model with **real options** that explains the cross-section of option prices

Real options matter for financial options

- Real options (ability to expand, launch new products, etc.) matter for the distributions of stock returns
- Firms with a lot of real options have the risk of cancelling their plans due to an economic slowdown
  - More intense when economy is in a boom and as the probability of exercising the real option is high $\Rightarrow$ **effect varies with aggregate state**
  - Stock has relatively limited upside and significant downside $\Rightarrow$ **generates a skew**
- Firms with few real options will not experience a sharp decline
- The conditional distribution of stock returns will differ among such firms
  - Option prices allow us to observe the distribution (Breeden and Litzenberger, 1978)

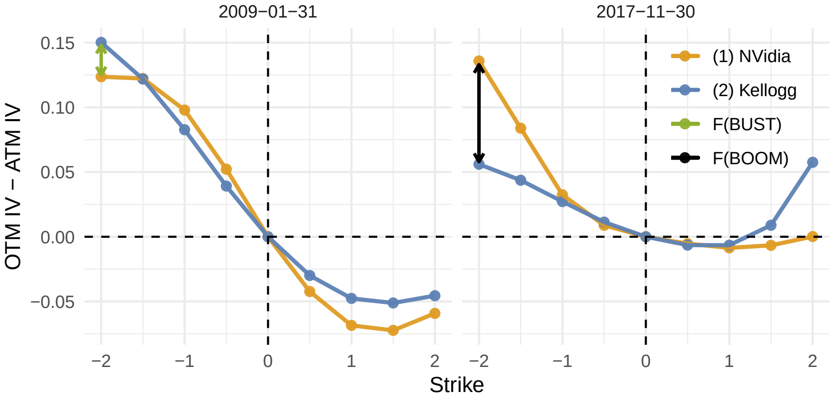
Results

1. I document that during booms OTM puts are relatively more expensive for growth firms than for value firms
   - ... but not in busts
2. I develop a production model with real options and aggregate risk consistent with this evidence
3. I use a structural corporate finance model to match both qualitatively and quantitatively the relative valuation of options in the cross-section
4. Show that real option model provides an explanation for recently proposed option based trading strategies based on firm fundamentals

Outline

1. Cross-section of implied volatilities
   - ⇒ Effect of book-to-market and leverage on equity options
   - ⇒ Effect heterogeneity in aggregate state
2. Continuous time model with real options
   - ⇒ Link to existing option pricing literature set in continuous time
   - ⇒ Show effect of real options on financial options
3. Discrete time model production model with debt
   - ⇒ Match the cross-sectional heterogeneity
4. Implications of continuous time model option strategies
   - ⇒ Returns of delta hedged option strategies within the model
   - ⇒ Rationalize the returns of recently proposed fundamental sorted portfolios

Growth and Value have different skews, example



- Compare implied volatilities for Kellogg (value) vs. NVidia (growth)
- ... for **bust** (Jan 2009) and **boom** (Nov 2017)
- Empirical strategy "estimates the arrows" as a function of firm characteristics and aggregate state

Empirical specification

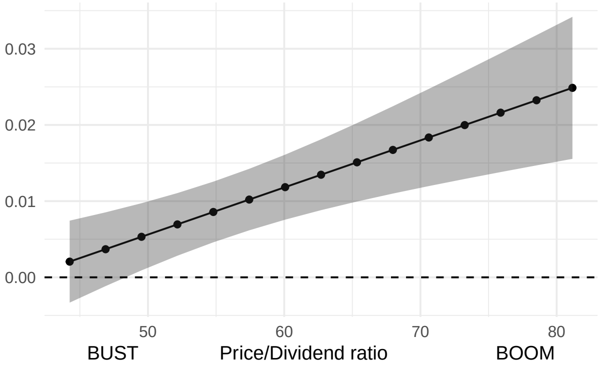
Specification that allows for heterogenous effect across the aggregate state

$$Skew_{i,t} = \alpha_t + \underbrace{(\beta^{MB} + \gamma^{MB} \cdot PD_t)}_{\text{Plotted on the next slide}} \cdot MB_{i,t} + \sum_X (\beta^X + \gamma^X \cdot PD_t) \cdot X_{i,t} + B' Z_{i,t} + \varepsilon_{i,t}$$

- PD – aggregate price-dividend ratio – state of the economy
- MB – market-to-book
- Other firm characteristics X: Leverage, Profitability, Investment, Size
    - Cross-sectionally ranked within industry and normalized between -1 and 1
- Controls Z: past realized return volatility and skewness

Systematic heterogeneity in skew across states

Plot $\hat{\beta}^{MB} + \hat{\gamma}^{MB} \cdot PD$ as a function of PD



- Consistent with the example: growth (high MB) have higher skew than value (low MB) in booms but not in busts

Real options model in words

Standard production model with real options (Back (2017), Gomes and Schmid (2010), ...)

- Firm starts with a fixed capital K with stochastic productivity x
- Has an option (only one) to expand its capital to a higher level
- Exercises its option when x reaches $\overline{x}$

Add time-varying price of cash flow risk λ

- Negatively correlated with the cash flow shock
- $\Rightarrow$ state dependent exercise boundary $\overline{x} = \overline{x}(\lambda)$
- $\overline{x} = \overline{x}(\lambda)$ is increasing and concave

Main channel: when productivity x falls firm value falls due to

- **Standard**: cash flow shock and correlated discount rate shock
- **Non-standard**: firm is pushed away from $\overline{x}(\lambda)$ that has a stronger effect for low λ due to concavity

Real options model in equations

- Start with capital K_0 + option to expand to K_1 by raising $K_1 - K_0$ of equity
- Given capital K cash flows are $x_t K^\alpha$ where

$$\frac{dx_t}{x_t} = \mu_x dt + \sigma_x dB_{x,t}$$

- Exogenous pricing kernel π_t and price of risk (similar to Wachter, 2013)

$$\frac{d\pi_t}{\pi_t} = -rdt - b\sqrt{\lambda_t}\sigma_x dB_{x,t}, \quad d\lambda_t = \kappa(\bar{\lambda} - \lambda_t)dt + \sigma_\lambda \sqrt{\lambda_t} dB_{\lambda,t}$$

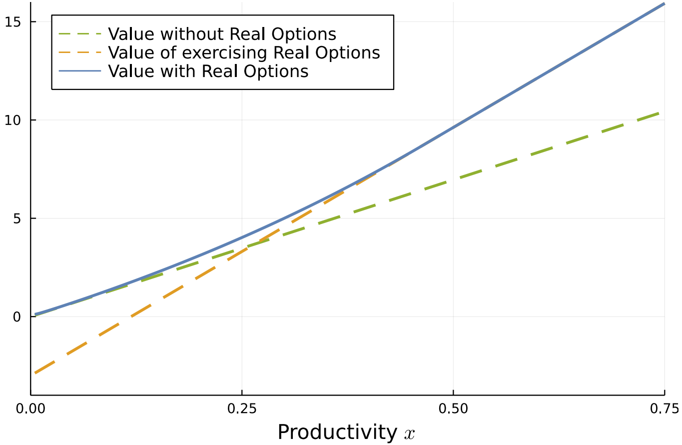
- **Young** firm value V^Y solves

$$V^Y(x_t, \lambda_t) = \left\{ V^M(x_t, \lambda_t) - (K_1 - K_0),\ x_t K_0^\alpha dt + E\left[\frac{\pi_{t+dt}}{\pi_t} V^Y(x_t + dx_t, \lambda_t + d\lambda_t)\right]\right\}$$

where **mature** firm value V^M solves

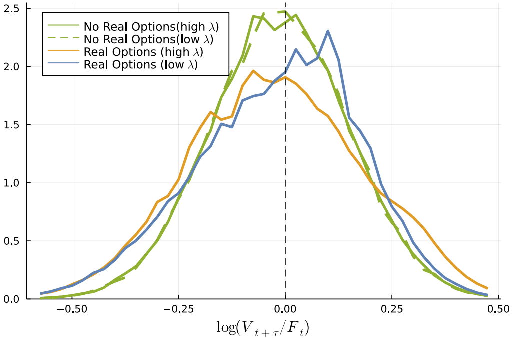
$$V^M(x_t, \lambda_t) = x_t K_1^\alpha dt + E\left[\frac{\pi_{t+dt}}{\pi_t} V^M(x_t + dx_t, \lambda_t + d\lambda_t)\right]$$

Model solution

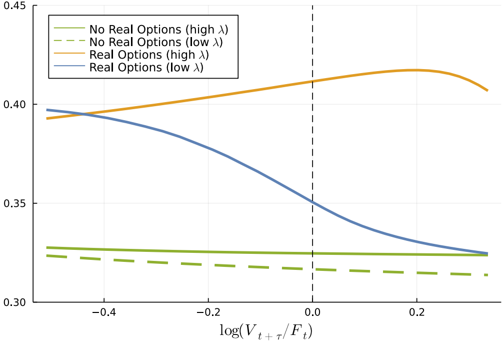


Real options ⇒ skewed distributions, skew in options

Distribution of firm value $\log(V_{t+\tau}/F_t)$



Implied volatility *IV*



- Value disribution for firm with no real options is not very sensitive to λ – i.e. whether the economy is in a boom or a bust (solid green vs dashed green)
- For a firm with real options, value distribution is close to symmetric in **booms** (orange) and is **negatively skewed** in **busts** (blue)

⇒ Leads to **skew** in implied volatility

Discrete time model

- Firm's problem

$$V_t(S_t) = \max_{K_{t+1}, B_{t+1}} F(d(S_t, K_{t+1}, B_{t+1})) + E\left[M(x_{t+1}|x_t)V(S_{t+1})|\, S_t\right].$$

  - State: $S = (K, B, x, y)$, x, y – aggregate and idiosyncratic productivities
  - SDF: $M(x_{t+1}|x_t)$: based on EZ utility of agent consuming $C(x) = e^x$
  - Equity issuance cost: $F(d) = d + (\chi_d + c_d d)\mathbb{I}_{d<0}$
  - Default/exit if $V(S_t) < 0$

- Dividend:

$$d(S_t, K_{t+1}, B_{t+1}) = \underbrace{e^{\beta_x x + y} K_t^{\alpha} - c_f}_{\text{Operating cash flows}} + \underbrace{\frac{B_{t+1}}{1+R} - B_t}_{\text{Net borrowings}} - \underbrace{\Phi(K_{t+1}, K_t)}_{\text{Investment costs}}$$

  - Asymmetric Φ following Zhang (2005) to generate variation in market-to-book

- Proceeds from borrowing following Begenau and Salomao (2018)

$$\underbrace{\frac{B_{t+1}}{1+R}}_{\text{Proceeds at } t} \equiv \underbrace{E\left[M_{t+1}\mathbb{I}_{V_{t+1}>0} B_{t+1}\right]}_{\text{Not default}} + \underbrace{E\left[M_{t+1}\mathbb{I}_{V_{t+1}=0} \min\{\theta(1-\delta)K_{t+1}, \overline{RC}\cdot B_{t+1}\}\right]}_{\text{Default}},$$

Model simulation and variable definition

Simulate 250 economies over 100 quarters. To mirror the data part

- For each state, solve for option prices and invert BSM to get implied volatilities
- Form right hand side variables to mirror data construction

$$MB = \frac{V-B}{K},\ Lev = \frac{B}{K},\ Prof = y,\ Inv = K_{t+1} - (1-\delta)K_t,\ Size = \log(V),$$

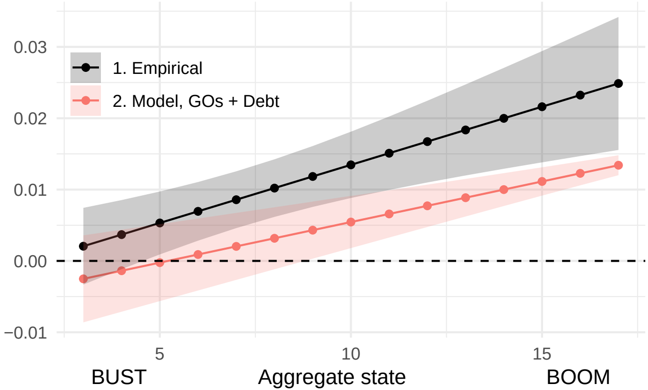
- aggregate state $= x$
- Normalize variables cross-sectionally

Estimate regression specification similar to data

$$Skew_{i,t} = (x \text{ FE}) + (\beta^{MB} + \gamma^{MB} \cdot x) \cdot MB_{i,t} + \sum_k (\beta^k + \gamma^k \cdot x) \cdot X_{i,t}^k + \varepsilon_{i,t}$$

Model fit, Market-to-Book

Compare $\hat{\beta}^{MB} + \hat{\gamma}^{MB} \cdot (\text{Aggregate state})$ in model vs. data



- Structural model captures the heterogeneity in skew across market-to-book

Delta-hedged trading strategy

Recent literature (Zhan et al, 2022) proposed highly profitable option strategies based on firm fundamentals

- Real options model allows to rationalize strategies based on book-to-market and profitability sorts

What is a delta-hedged option strategy?

- Consider a long position in a call option combined with a short position in the underlying asset equal to delta of this option
  - Delta – sensitivity of option price to underlying
  - Such position is called delta neutral
- As the price of the underlying moves its delta changes $\Rightarrow$ position is no longer delta neutral
- The trader needs to adjust its position in the underlying asset

Delta hedged option strategies are widely traded

- Market makers hedged their exposures to movements in underlying asset
- Volatility traders use delta hedging to *purify* their exposure to volatility risk – unique risk embedded in options

Delta hedged returns in real options model

Bakshi and Kapadia (2003): expected profits of a delta hedged strategy are related to variance risk premium

$$E[Profit] = \int_t^{t+\tau} E_t\left[\frac{\partial C}{\partial \sigma}\lambda(\sigma)\right] du, \quad \lambda(\sigma) = cov_t\left(-\frac{d\pi_t}{\pi_t}, d\sigma_t\right)$$

I derive a similar result in the real options model

Proposition

Expected profits from a continuously delta hedged long option position are

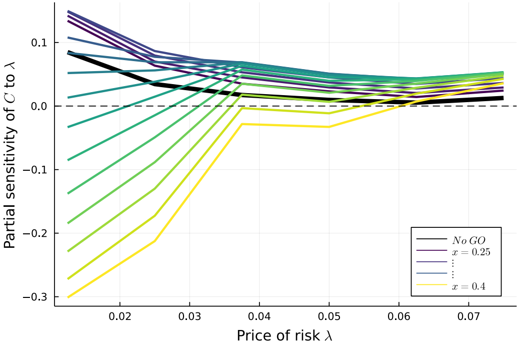
$$E[Profit] = \int_t^{t+\tau} \sigma_x \sigma_\lambda b \rho E_t\left[\frac{\partial \widetilde{C}}{\partial \lambda}\lambda_u\right] du, \quad \frac{\partial \widetilde{C}}{\partial \lambda} \equiv \frac{C}{\partial \lambda} - \frac{\partial F/\partial \lambda}{\partial F/\partial x} \cdot \frac{\partial C}{\partial x}$$

where $F(x,\lambda) \Rightarrow C(F(x,\lambda),\lambda) = \widetilde{C}(x,\lambda)$

- If only one *stochastic* state $(\sigma_\lambda = 0) \Rightarrow E[Profit] = 0$
- If 2nd stochastic state (λ) is not priced $(\rho = 0) \Rightarrow E[Profit] = 0$

Direct sensitivity

The direct sensitivity of option price C to price of risk λ, $\partial \tilde{C}/\partial\lambda$



Delta-hedged profits:

$$E[Profit] = \int_t^{t+\tau} E_t\left[\frac{\partial \widetilde{C}}{\partial \lambda}\sigma_x\sigma_\lambda b\lambda_u\rho\right]$$

- For a given state λ:

$$\underbrace{\rho}_{<0} \times \underbrace{\left[\left(\frac{\partial \tilde{C}}{\partial \lambda}\right)_{Value} - \left(\frac{\partial \tilde{C}}{\partial \lambda}\right)_{Growth}\right]}_{>0} < 0$$

$$\Rightarrow E[Profits]_{Value} < E[Profits]_{Growth}$$

- Stronger when price of risk λ is low

Conclusion

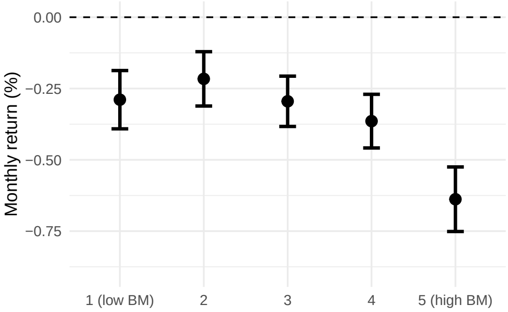
- I show that there is a state-dependent cross-sectional heterogeneity in equity options
- I provide an theoretical framework based on real options to understand the observed relationship
- I show that a dynamic production model can match the evidence both qualitatively and quantitatively
- I rationalize the recently proposed highly profitable option strategies based on cross-sectional sorts on firm fundamentals

Trading strategy: construction

At the end of each month

1. Rank companies by book leverage $\Rightarrow$ keep only $<$ median
2. Rank companies within industry by book-to-market/profitability $\Rightarrow$ form 5 bins
3. Construct a straddle (call + put) for an $\approx$ 3 month maturity for each bin

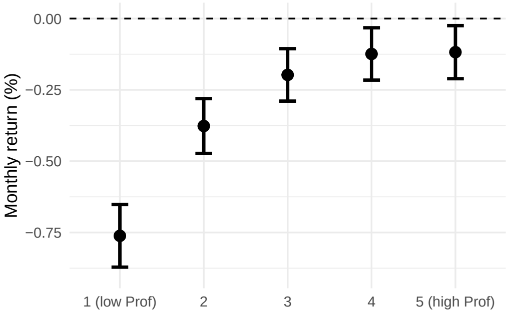
Book-to-Market bins



	Low BM	2	3	4	high BM	HML
mean (%)	-0.289	-0.216	-0.295	-0.364	-0.638	-0.349
stdev (%)	1.80	1.68	1.56	1.66	2.00	1.16
Sharpe	-0.556	-0.446	-0.657	-0.761	-1.11	-1.04

Operating profitability bins

Directly from Zhan et al (2022)



name	Low Prof.	2	3	4	High Prof	HML
mean (%)	-0.762	-0.377	-0.198	-0.124	-0.118	0.644
stdev (%)	1.94	1.69	1.62	1.62	1.64	0.971
Sharpe	-1.36	-0.770	-0.422	-0.265	-0.249	2.30