Asset Pricing Notes. Chapter 11: Imperfect Risksharing

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1 Incomplete Markets

1.1 Grossman and Shiller (1982)

This classic argument implies that individual income shocks and hence idiosyncratic movements in consumption don't have any effect in the risk premia and what is relevant for risk premia is the covariance of returns with **aggregate** consumption. Start from pricing equation in excess return form for agent k:

$$0 = E_t\left[(R_{i,t+1} - R_{j,t+1})\frac{U'(C_{k,t+1})}{U'(C_{k,t})}\right]$$

Now use approximation of marginal utility $U'(C_{k,t+1})$ around time t consumption

$$U'(C_{k,t+1}) = U'(C_{k,t}) + U''(C_{k,t})(C_{k,t+1} - C_{k,t})$$

This approximation becomes accurate as the time interval shrinks. Thus we have

$$\begin{aligned} 0 &= E_t\left[(R_{i,t+1} - R_{j,t+1})\frac{U'(C_{k,t}) + U''(C_{k,t})(C_{k,t+1} - C_{k,t})}{U'(C_{k,t})}\right] \\ &= E_t\left[(R_{i,t+1} - R_{j,t+1})\left(1 - A_{kt}\Delta C_{k,t+1}\right)\right] \end{aligned}$$

where we used the definition of absolute risk aversion $A_{kt} = -\frac{U''(C_{k,t+1})}{U'(C_{kt})}$. Rearrange this expression a bit to get

$$\begin{aligned} E_t[R_{i,t+1} - R_{j,t+1}] &= A_{kt}E_t[(R_{i,t+1} - R_{j,t+1})\Delta C_{k,t+1}] \\ \frac{1}{A_{kt}}E_t[R_{i,t+1} - R_{j,t+1}] &= A_{kt}E_t[(R_{i,t+1} - R_{j,t+1})\Delta C_{k,t+1}] \end{aligned}$$

Sum this expression across agents k

$$\begin{aligned} \left(\sum_k \frac{1}{A_{kt}}\right)E_t[R_{i,t+1} - R_{j,t+1}] &= E_t\left[(R_{i,t+1} - R_{j,t+1})\left(\sum_k \Delta C_{k,t+1}\right)\right] \\ E_t[R_{i,t+1} - R_{j,t+1}] &= \left(\sum_k \frac{1}{A_{kt}}\right)^{-1} E_t\left[(R_{i,t+1} - R_{j,t+1})\Delta C_{t+1}\right] \end{aligned}$$

In this expression only covariance with aggregate consumption matters for asset prices. Thus the asset prices are same as in the model with a representative investor that has Absolute risk aversion equals to harmonic mean of all agents risk aversions.

1.2 Constantinides and Duffie (1996)

They arrive at a different conclusion. They start by noting that if we average individual agents SDF we will get a valid SDF (because of the linearity of the pricing equation). Define the cross section mean as E^* and cross sectional variance as var^*. Then such cross-sectionally averaged SDF is

$$M^*_{t+1} = E^*_{t+1}\left[\left(\frac{C_{k,t+1}}{C_{kt}}\right)^{-\gamma}\right]\delta$$

Assuming cross sectional lognormality we get the foloowing (in the same way as assuming lognormal distribution of returns):

$$m^*_{t+1} = \log(\delta) - \gamma E^*_{t+1}\Delta c_{k,t+1} + \frac{\gamma^2}{2} var^*_{t+1}(\Delta c_{k,t+1})$$

The log SDF depends on aggregate consumption growth ($E^*_{t+1}\Delta c_{k,t+1} = \Delta c_{t+1}$) but also it depends on cross sectional dispersion on consumption growth ($\frac{\gamma^2}{2} var^*_{t+1}(\Delta c_{k,t+1})$).

Now consider an economist that observes aggregate consumption $E^*_t c_{k,t+1}$ at each moment t. If he decides to ignore heterogeneity in consumption growth and assume existence of a representative agent he may construct an **incorrect** SDF as

$$M^{RA}_{t+1} = \delta\left(\frac{E^*_{t+1}C_{k,t+1}}{E^*_t C_{k,t}}\right)^{-\gamma}$$

Now consider the log of this SDF

$$\begin{aligned}
m^{RA}_{t+1} &= \log(\delta) - \gamma(\log(E^*_{t+1}C_{k,t+1}) - \log(E^*_t C_{k,t})) \\
&= \log(\delta) - \gamma\left(E^*_{t+1}c_{k,t+1} + \frac{1}{2}Var^*_{t+1}(c_{k,t+1}) - E^*_t c_{k,t} - \frac{1}{2}Var^*_t(c_{k,t})\right) \\
&= \log(\delta) - \gamma\left(E^*_{t+1}c_{k,t+1} - E^*_t c_{k,t}\right) - \gamma\left(\frac{1}{2}Var^*_{t+1}(c_{k,t+1}) - \frac{1}{2}Var^*_t(c_{k,t})\right) \\
&= \log(\delta) - \gamma\left(E^*_{t+1}c_{k,t+1} - E^*_t c_{k,t}\right) - \gamma\left(\frac{1}{2}Var^*_{t+1}(c_{k,t} + \Delta c_{k,t+1}) - \frac{1}{2}Var^*_t(c_{k,t})\right)
\end{aligned}$$

Now assume that consumption growth $\Delta c_{k,t+1}$ is cross sectionally uncorrelated with c_{kt}

$$\begin{aligned}
m^{RA}_{t+1} &= \log(\delta) - \gamma\left(E^*_{t+1}c_{k,t+1} - E^*_t c_{k,t}\right) - \frac{\gamma}{2}Var^*_{t+1}(\Delta c_{k,t+1}) \\
&= \log(\delta) - \gamma\left(E^*_{t+1}\Delta c_{k,t+1}\right) - \frac{\gamma}{2}Var^*_{t+1}(\Delta c_{k,t+1})
\end{aligned}$$

What is the difference between the correct m^*_{t+1} and an *incorrect* m^{RA}_{t+1}

$$\begin{aligned}
m^*_{t+1} - m^{RA}_{t+1} &= \frac{\gamma^2}{2}var^*_{t+1}(\Delta c_{k,t+1}) + \frac{\gamma}{2}Var^*_{t+1}(\Delta c_{k,t+1}) \\
&= \frac{\gamma(\gamma+1)}{2}var^*_{t+1}(\Delta c_{k,t+1})
\end{aligned}$$

Term $var^*_{t+1}(\Delta c_{k,t+1})$ can have a non-zero mean and non-zero variance helping to explain the equity premium. If $var^*_{t+1}(\Delta c_{k,t+1})$ increases in downturns, meaning that heterogeneity increases in downturns then the true SDF m^*_{t+1} will be more procyclical than *incorrect* SDF m^{RA}_{t+1} inferred from the data.

The difference with Grossman Shiller conclusion comes from approximation accuracy. In continuous time term $var^*_{t+1}(\Delta c_{k,t+1})$ is deterministic since it is a quadratic variation. Hence, it is not time varying meaning that the difference between the correct and incorrect SDFs stays constant and doesn't affect the risk premium. This also means that gradual changes in heterogeneity don't help to generate equity premium and we need to think about disasters that suddenly lead to reallocation of wealth and, as a result, consumption growth to explain the equity premium.

1.3 Market Design and Incomplete Markets

Athanasoulis and Shiller (AS 2000) consider an economy with agents with different endowments and different risk aversions. They consider a problem of a social problem that can design contract that are income swaps and ask what are the optimal contract to maximize welfare (generate the best risk sharing) subject to a constraint on number of contracts. The social planner faces the following trade off

1. When all risk aversions are the same, optimal contract is orthogonal to the world portfolio and swaps idiosyncratic endowment fluctuations
2. When agents differ in their risk aversion, it is efficient to let the more risk tolerant agent to insur the more risk averse agent. Hence, in this case the optimal contract will have a positive weight on the market portfolio.

Simsek (2013) argues that when agents have heterogeneous beliefs they can use new markets not only to share risks but also to speculate. If the latter effect dominate this can lead to increase in cross sectional volatility of consumption rather than a reduction of it.

2 Default

2.1 Punishment By Exclusion

If an agent has the ability to default in different states of the world this will limit their ability to issue claims and thus the risk sharing may be imperfect. Stronger punishment of default $\implies$ lower probability of default $\implies$ larger positions $\implies$ better risk-sharing. Alvarez and Jermann (2000) present a model where punishment for default is a permanent exclusion from the market, i.e. forcing an agent into autarky.

In their model, they consider a constrained planner's problem that should take incentivize agents not to default: $U(\{c_j\}) \geq U(\{e_j\})$.

Main Takeaways

- They derive reduced form condition to see whether any risk sharing is possible (autarky is not constrained-efficient). The conditions when risk sharing is not possible are
  1. Low β: if agents don't value the future, can't create incentives not to default
  2. Small risk aversion: agents don't care that much about higher volatility of marginal utility in autarky compared to risk sharing allocations
  3. Low variance of idiosyncratic shocks: the same
  4. Transition matrix for states is close to identity: shocks are persistent. If shocks are persistent, then after a good realization the agent may choose to default because he expects to stay in the same state for a long time.
- Increase in cross section income risk can actually **reduce** consumption risk because it makes it easier to punish default which improves risk sharing
- In some cases, securities prices are higher in the presence of solvency constraints since there is a limit ability to short securities.

2.2 Punishment by Seizure of Collateral

- Chien and Lustig (2010) argue that punishment by seizure of collateral is more empirically plausible than punishment by exclusion. In this case agents are prevented from issuing contingent claims in excess of the value of their collateral. In this model value of collateral becomes a state variable. Bad shock $\implies$ value of collateral falls $\implies$ risk-sharing worsens $\implies$ asset prices may be affected through heterogeneity channel of Constantinides and Duffie (1996): equity premium should increase when value of collateral falls as there is more cross sectional variation in consumption growth hence in marginal utilities.

- Difficulty with all these models of default is that agents want to default after a good shock. Thus we can get the following dynamic: good shock happens and consumption increases $\Longrightarrow$ agents default and walk away from their collateral $\Longrightarrow$ collateralized wealth declines $\Longrightarrow$ negative correlation between consumption and collateralized wealth.